Computational Cognitive Science



Lecture 14: Iterated learning (continued)



 $\begin{bmatrix} n=1 \\ n=2 \\ n=3 \\ n=4 \\ n=5 \\ n=6 \\ n=7 \\ n=8 \\ n=7 \\ n=8 \\ n=9 \\ n=9 \\ n=9 \\ n=0 \\ n$



We started looking a little about how to study things that change over time. As a first stab, we began looking at conceptual change /evolution over time



Ancient Greece/Rome: - marriages arranged - affairs (for men) okay, including with young boys - not usually for love



medieval times:

- still usually economic
- often involved dowries
- women were property
- church involved more



19th century times:

- occasionally for love
- often not cross-racial
- women sometimes can keep property



1950s etc

- often for love
- cross racial sometimes ok
- women "in the home"



today:

- usually for love
- cross-racial okay
- same-sex sometimes ok
- women wield much more economic power

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- This is modeled as chains of learners who pass each other information, and are individually Bayesian in how they learn from the previous one



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But how (and in what ways) are these results dependent on certain assumptions we had to make?

Today's plan

- Evidence for conceptual evolution
 - Inevitable given noisy transmission
 - Historical record
 - Cultural variation
- A model of conceptual change over time
 - Iterated learning model: basic idea
 - Mathematical proof and corresponding intuition
- Experimental evidence for iterated learning models
 - Function learning
 - Language
- Limitations and extensions to the iterated learning model
 - changing learner
 - changing producer
 - changing how hypotheses map onto the world

- The iterated learning model as it stands now contains a lot of different assumptions. We can explore the effect of each of them!
 - What learners do with the data (i.e., how they infer hypotheses)
 - How producers produce the data
 - How hypotheses relate to the events in the world

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So far iterated learning assumes that learners select a grammar by sampling it from the posterior distribution over hypotheses

$$P(h_{n+1}|x_n, y_n) = \frac{P(y_n|x_n, h_{n+1})P(h_{n+1})}{\sum_{h \in \mathcal{H}} P(y_n|x_n, h)P(h)}$$

An alternative is that learners select the hypothesis that is most probable. This is called MAP learning (maximum a posteriori)



If people are MAP learners, how does this change the convergence behaviour of the chain?

MAP learning

- ▶ If people are MAP learners, it is now much more complicated.
- Overall, the distribution of hypotheses still reflects the ordering of hypotheses in the prior, but differences are magnified -- the *a priori* most likely hypothesis will be overrepresented



This means the same stationary distribution can result from different priors



MAP learning

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- Overall, the distribution of hypotheses still reflects the ordering of hypotheses in the prior, but differences are magnified -- the *a priori* most likely hypothesis will be overrepresented



- This means the same stationary distribution can result from different priors
- In addition, changing transmission factors (like the amount of data) can result in convergence to a different stationary distribution

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 - What learners do with the data (i.e., how they infer hypotheses)
 - How producers produce the data
 - How hypotheses relate to the events in the world

Changing assumptions about the producer

Originally we assumed that people produce data by drawing it at random from their inferred hypothesis



 $P(y_{n+1}|x_{n+1},h_{n+1})$

But in real life, at least sometimes, people can actually just describe their hypothesis.

> formally, we just assume each agent's prior is the posterior of the previous agent

 $P(h_n|d_n)$

Changing assumptions about the producer

Originally we assumed that people produce data by drawing it at random from their inferred hypothesis



But in real life, at least sometimes, people can actually just describe their hypothesis.

just the same as assuming it is one agent who saw all of the data over time

$$P(h_n|d_n)$$

formally, we just assume each agent's prior is the posterior of the previous agent

Changing assumptions about the producer

Experimental test: vary what information people are allowed to pass along

Control: typical frequency learning experiment



Posterior passing: at

end, participants can describe what they thought the function was This seemed like when the training item is really high or really low, the test is at the top. If the training is in the middle the test is at the bottom.

Posterior passing changes the distribution!



Interesting implications for cultural evolution -- our ability to explicitly pass information along (by teaching) effectively makes humanity able to learn from hundreds or thousands of years of experience, rather than one lifetime. This is called the cultural rachet.

- The iterated learning model as it stands now contains a lot of different assumptions. We can explore the effect of each of them!
 - What learners do with the data (i.e., how they infer hypotheses)
 - How producers produce the data
 - How hypotheses relate to the events in the world

Suppose these are the events in the world...



As we've seen many times, some ways of classifying these events seem natural and others not-so-natural



More natural

Less natural

Yet convergence to the prior suggests that the stationary distribution should occur independently of what the actual structure of events in the world is!



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This seems wrong.

• Learning step: learner n+1 sees x_n (from previous person) and computes a posterior distribution over h_{n+1} according to Bayes' Rule

$$P(h_{n+1}|x_n, y_n) = \frac{P(y_n|x_n, h_{n+1})P(h_{n+1})}{\sum_{h \in \mathcal{H}} P(y_n|x_n, h)P(h)}$$

- This form of the equation follows from the assumption that each hypothesis h makes no assumptions about which events are more likely
- But, that might be silly in many situations. We might instead presume that hypotheses carry with them some assumptions about the event structure in the world:

$$P(h_{n+1}|x_n, y_n) = \frac{P(y_n|x_n, h_{n+1})P(h_{n+1}|x_n)}{\sum_{h \in \mathcal{H}} P(y_n|x_n, h)P(h|x_n)}$$

Rederiving with the new equation...

The resulting Markov Chain has a stationary distribution that now converges to $\pi(h) = \sum_{x} P(h|x)Q(x)$

$$\begin{aligned} \pi(h_{n+1}) &= \sum_{h_n} P(h_{n+1}|h_n)\pi(h_n) \\ &= \sum_x \sum_y \sum_{h_n} P(h_{n+1}|x,y)P(y|x,h_n)Q(x)\pi(h_n) \\ &= \sum_x \sum_y \sum_{h_n} P(h_{n+1}|x,y)P(y|x,h_n)Q(x)\sum_x' P(h_n|x')Q(x') \\ &\approx \sum_x \sum_y \sum_{h_n} P(h_{n+1}|x,y)P(y|x,h_n)Q(x)P(h_n|x) \\ &= \sum_x \sum_y P(h_{n+1}|x,y)Q(x)\sum_{h_n} P(y|x,h_n)P(h_n|x) \\ &= \sum_x \sum_y P(h_{n+1}|x,y)Q(x)P(y|x) \\ &= \sum_x Q(x)\sum_y P(h_{n+1}|x,y)P(y|x) \\ &= \sum_x Q(x)P(h_{n+1}|x) \\ &= \pi(h_{n+1}) \end{aligned}$$
 Must assume that
$$P(h|x) \approx E_{Q(x')}[P(h|x')] = \sum_{x'} P(h|x')Q(x') \\ \text{which basically comes down to assuming that } x \text{ is a representative draw from } Q(x) \end{aligned}$$

What does this mean?

- Cultural evolution should converge on the languages / concepts that are likely given the structure of the world and the posterior distribution over those languages / concepts
 - Incorporates the structure of the world
 - Prior probability of concepts / languages
 - Also their likelihood given the data produced
 - Previous experiments didn't really manipulate event structure (x)... only the x,y pairing (as in function learning).
 - A prediction here is that manipulating event structure only should still strongly affect what is learned!

Event structure, as before, is a set of meanings, but this time simpler: 36 possible, varying on two dimensions



- Events are paired with a label (for first person, it's random)
- After training on these, the person is shown events and has to generate the label themselves
- The next person is given the previous person's labels to use as their training data

Event structure, as before, is a set of meanings, but this time simpler: 36 possible, varying on two dimensions











\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_2	\mathbf{W}_2	\mathbf{W}_2
\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_2	\mathbf{W}_2	\mathbf{W}_2
\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_2	\mathbf{W}_2	W ₂
\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_2	\mathbf{W}_2	W ₂
\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_2	W ₂	W ₂
\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_2	\mathbf{W}_2	W ₂

\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	W_1	W_1	\mathbf{W}_1
\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1
\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1	\mathbf{W}_1
\mathbf{W}_2	\mathbf{W}_2	\mathbf{W}_2	\mathbf{W}_2	\mathbf{W}_2	\mathbf{W}_2
W ₂ W ₂					

First, we can look at the final languages in each chain:





= KUM

= HEC

= LRP

Colour

= BL

= ROK

= ROOK

= NAF

= PUG

= DIL

Quantify with the adjusted Rand Index

(compared to canonical size and canonical colour; 1=identical; 0=random)

	Canonical Size	Canonical Colour
Control	-0.0204	0.0618
Size	0.704	0.079
Colour	0.065	0.696

The full chains tell the same story - gradual evolution to match the structure of the world

Control



The full chains tell the same story - gradual evolution to match the structure of the world







- We can model conceptual change / evolution as chains of learners who pass each other information, and are individually Bayesian in how they learn from the previous one
- The main prediction, that the stationary distribution of the chain reflects (only) prior probability, was borne out experimentally
- Changing the assumptions changes the results in interesting ways





Computational Cognitive Science



log number of occurrences of word

Lecture 15: Sequential learning with n-grams



Bigram frequencies

So far we've been looking at understanding how concepts can change over a long time span, based on the sequence of generations or states that change goes through



- So far we've been looking at understanding how concepts can change over a long time span, based on the sequence of generations or states that change goes through
- But much in day-to-day life we are faced with sequences that occur on a much smaller time scale, and we need to learn about them



what is that melody?

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- But much in day-to-day life we are faced with sequences that occur on a much smaller time scale, and we need to learn about them



what is she going to say next?

- So far we've been looking at understanding how concepts can change over a long time span, based on the sequence of generations or states that change goes through
- But much in day-to-day life we are faced with sequences that occur on a much smaller time scale, and we need to learn about them



where is the monster going to go next?

- So far we've been looking at understanding how concepts can change over a long time span, based on the sequence of generations or states that change goes through
- But much in day-to-day life we are faced with sequences that occur on a much smaller time scale, and we need to learn about them



is he going to make his next goal?

- So far we've been looking at understanding how concepts can change over a long time span, based on the sequence of generations or states that change goes through
- But much in day-to-day life we are faced with sequences that occur on a much smaller time scale, and we need to learn about them



what is the plan for the day?









All of these involve learning what kinds of *sequences* of actions there are, and which actions tend to follow which others

We can therefore apply similar models to learning all of them, although the specific questions we care about in each will differ

Plan for the rest of the lectures

- ▶ Today: a simple model for sequence learning (*n*-grams)
 - Description of the approach
 - Application to natural language processing
 - The problem of overfitting
- ▶ Tomorrow 1: applications of *n*-gram models
 - A solution to the problem of overfitting
 - Word segmentation
 - Nonadjacent learning
 - What about more complex structure?
- ▶ Tomorrow 2: extending *n*-grams (HMMs)
 - Computing likelihood of observations
 - Inferring the hidden state sequence
 - Finding the best HMM (if time)

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the itsy bitsy spider went up the water spout...

The techniques we will be talking about are a simplified version of techniques used by many companies and researchers who want to use and manipulate text



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harry porter				
oks	Book	Videos	Images	Web
)	Во	Videos	Images	Web

Did you mean: harry potter

the itsy bitsy spider went up the water spout...

The techniques we will be talking about are a simplified version of techniques used by many companies and researchers who want to use and manipulate text

$\overrightarrow{\mathbf{x}}$	Ashley Ogletree	Boner Extension Is Possible And Safe lea
${\sim}$	「黄金のKABU手法」小野寺	あなたが「株」で損をしてる原因を教えまし
\$	小野寺典夫 mag2 0000258547	あなたが「株」で損をしてる原因を教えまし
*	小野寺典夫 mag2 0000258547	「黄金のKABU手法」(公式):「1万時間
*	「黄金のKABU手法」小野寺	「黄金のKABU手法」(公式):「1万時間
公	Barack Obama	Hey, Amy - The top Republican in the House
☆	International Work-Confe.	Time Series Conference - ITISE 2014, Gran

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the itsy bitsy spider went up the water spout...

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- why is Australia so Google Search
- why is australia so expensive
- \bigcirc why is australia so hot
- why is australia so great
- \circ why is australia so dry
- 🔍 why is australia so boring

why is America so

- why is America so Google Search
- igsirin why is america so stupid
- why is america so religious
- why is america so violent
- $^{ extsf{Q}}$ why is america so rich
- $^{ ext{Q}}$ why is america so cheap

the itsy bitsy spider went up the water spout...

- The techniques we will be talking about are a simplified version of techniques used by many companies and researchers who want to use and manipulate text
- Cognitive scientists are interested in the same tools, because people may rely on something like them to solve similar problems



- Data = a corpus (plural: corpora), which is a (set of) text files used as input to your algorithm
- In cognitive science, where we're interested in the data people actually hear, it is often transcribed speech (often to children)

yuwanttusiD6bUk
&nd6d0gi
yuwanttulUk&tDIs
lUk&tDIs
h&v6drINk
tekItQt

```
you want to see the book?
and a doggie!
you want to look at this?
look at this!
have a drink
take it out
```

Corpus of child-directed speech transcribed into an ASCII version of phonetic notation



- Data = a corpus (plural: corpora), which is a (set of) text files used as input to your algorithm
- In cognitive science, where we're interested in the data people actually hear, it is often transcribed speech (often to children)
- In NLP, it is more often just gigabytes of text documents from the web, samples from articles / books / etc





There are a lot of hidden complexities involved in getting corpora to the point that we can get useful information about the language from them -- we'll skip over a lot of these



Homographs

saw vs saw

(in spoken corpora, homophones)

Basic problems in language modelling

What sequences of words are allowable or frequent?

hey! how are you?

pickled crysanthemums

• Given a sequence, what word(s) would you expect to come next?

I don't want to date you, I'd rather just be _____

Models called n-gram models are among the simplest ways to answer both of these problems (as well as in many non-language contexts as well, as we'll see tomorrow)

I'd rather just be _____

Wn-3 Wn-2 Wn-1 Wn

Essentially, we want to estimate the following probability function

 $p(w_n|w_1, ..., w_{n-1})$

1-gram	2-gram	3-gram	
(unigram)	(bigram)	(trigram)	
$p(w_n)$	$p(w_n w_{n-1})$	$p(w_n w_{n-1},w_{n-2})$	

Wn-3 Wn-2 Wn-1 Wn

Essentially, we want to estimate the following probability function

 $p(w_n|w_1, ..., w_{n-1})$

I'd rather just be *friends*

1-gram	2-gram	3-gram	
(unigram)	(bigram)	(trigram)	
P (friends)	P (friends be)	P (friends just be)	

Google labs Books Ngram Viewer



https://books.google.com/ngrams



(aliak on line/label for feaue)



You can also use these models to generate sentences rather than just predict them

Bigram

✓ Don't you get on the front of the bathroom?

- ✓ Ernie hesitated to Father Feeney, for me he said, "Come sit", he should bear the soul.
- ✓ And Bill Doolin and known magic might be as training?
- ✓ The previous night, means by the initial cost increases, the official representative government in address for manned bombers to gallop was Roman nose on a million of a couple of the book of purpose we fail to a cowhand was heard.

You can also use these models to generate sentences rather than just predict them

Trigram

- ✓ It wasn't just Japan, either, because the rain came more heavily, and men in an involuntorial fashion.
- ✓ These two were blacked out, as it did when Eisenhower appointed him Ambassador to Moscow.
- ✓ On chemical grounds it should be remitted with your naked eye at night in South Africa, beats it by taking the raw sewage into the Austrian branch of Multnomah Bank, also was ordered by a public servant but not much to remove restrictions that, it requires only written notice that he was glad the fat that extended beyond himself, something which, consequently, he was counting on the cheek.

You can also use these models to generate sentences rather than just predict them

4-gram

- ✓ A Yankee sergeant gave the following description of his sweetheart: "My girl is none of your business", she said, then turned and attacked Morgan who became greatly outnumbered and had to fasten it on someone.
- ✓ When words can be used in almost any classroom today can be found amongst us without a correct version of the welfare state in England still allows wild scope for all kinds of problems for which she was moving, her method for keeping an escort from departing too early was unique.
- ✓ But, again, we have no clocks and nobody cares enough to count days or to make calendars and there's not much climate here, so none of us knew anything but filth and poverty.

You can also use these models to generate sentences rather than just predict them

5-gram

- ✓ Nevertheless, it remained one of the great singers of our time but she is one of the best-gaited pacers on the grounds.
- ✓ These are few and seemingly disjointed data, but they illustrate the important fact that fundamental alterations in conditioned reactions occur in a variety of expressions in your country denied not only the existence of this conflict but it was elaborated even further with an incredible semantic dexterity.
- ✓ The queen afterward keeps incubating and guarding her eggs like a mother hen, taking a sip from time to time in ratable proportions, on account of the crazy tourists.

- ▶ For both generation and prediction, higher *n* is better!
- Both are extremely straightforward given the n-gram probabilities

Two kinds of probabilities

1. Probability of a word or series of words $p(w_1,...,w_n)$

2. Probability of a word given a previous word or series of words $p(w_n|w_1,...w_{n-1})$

The equations are distinct (except in the unigram case)



Sequence learning is an important problem in cognition, and language is a clear example of when this is relevant





Summary

- Sequence learning is an important problem in cognition, and language is a clear example of when this is relevant
- n-gram models, which calculate the probability of an item given the previous n-1 items, are widely used in natural language processing to address this problem.

9 why is Australia so

- 🔍 why is Australia so Google Search
- \circ why is australia so expensive
- 🔍 why is australia so hot
- \circ why is australia so great
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Additional references (not required)

N-gram models

▶ Manning, C., & Schutze, H. (1999). Foundations of statistical natural language processing. Chapter 5: 191-203